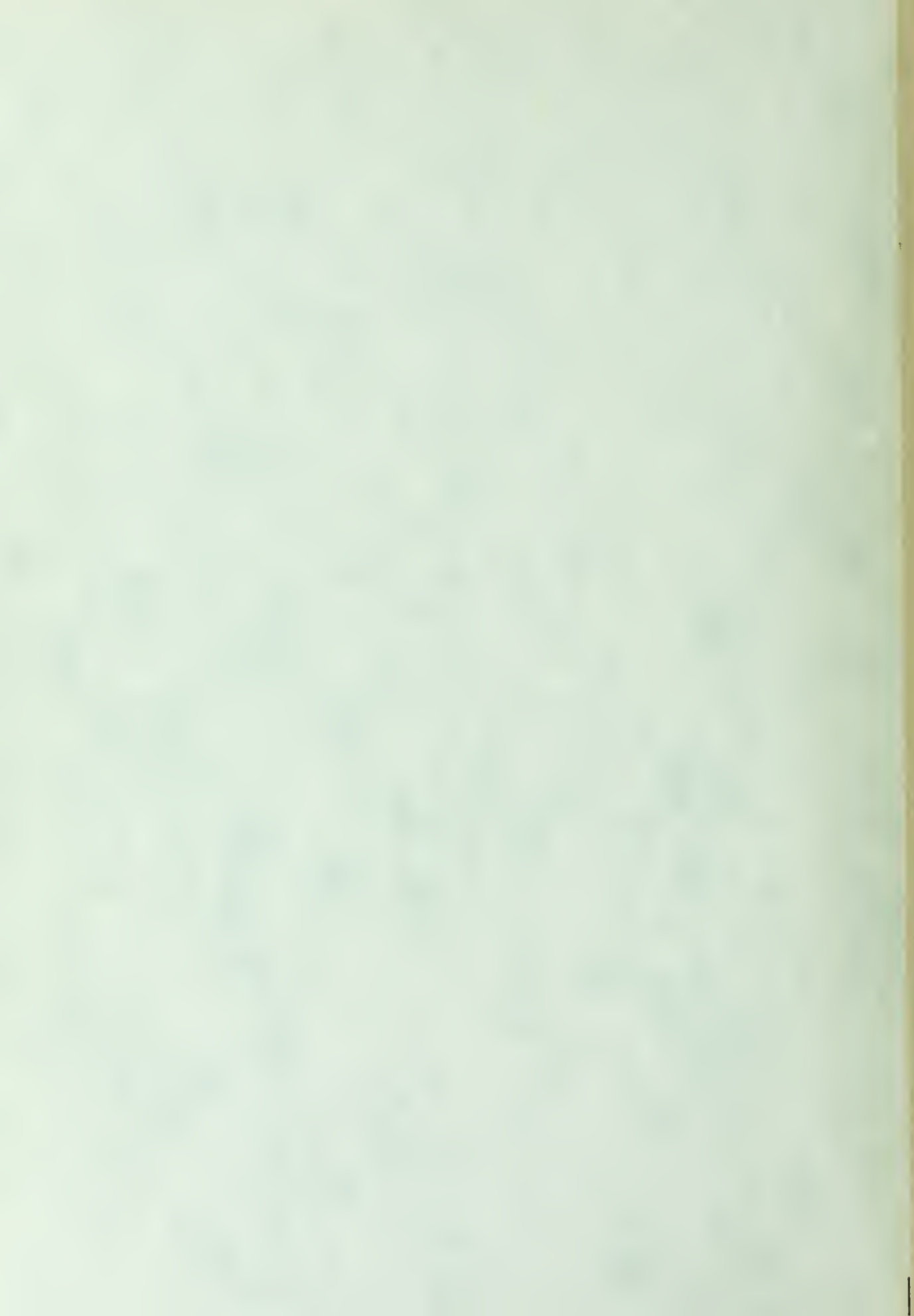A LOW IMPEDANCE HIGH CURRENT ULTRASONIC
SIGNAL SOURCE FOR FERRORESONANCE STUDY.

by

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United States
Naval Postgraduate School

THESIS

A LOW IMPEDANCE HIGH CURRENT
ULTRASONIC SIGNAL SOURCE
FOR FERRORESONANCE STUDY

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September 1970

A Low Impedance High Current
Ultrasonic Signal Source for Ferroresonance Study

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1970

ABSTRACT

In ferroresonant mode, a transformer is saturated and requires very high currents from the source. Previous studies of ferroresonance could not be performed effectively because of high internal impedance of the source. Introduction of transformers to improve these shortcomings would have also introduced undesirable inductive effects. In this thesis, a transistor voltage source with low internal impedance and high current delivering ability is designed and tested. The source is used to investigate an unsymmetrical mode of series resonance. Resonant circuit behavior is both qualitatively and quantitatively described.

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I. INTRODUCTION

Until the advent of high power transistors, obtaining low internal impedance signal sources with high current handling ability was virtually impossible in the ultrasonic region. With the expected frequency of ferroresonance in the neighborhood of 15 kilohertz to 25 kilohertz, the use of transformers for impedance transformation and current multiplication is impractical. The transformer inductance would limit rate-of-current changes when the ferroresonant core saturates and hysteresis effects would introduce non-linear harmonic distortion.

This circuit uses silicon transistors with both high current handling ability and high current gains while maintaining very wide frequency response. The source displays very low impedance, on the order of .25 ohm and delivers approximately 5 amperes of peak-to-peak signal current into a .1 ohm load. Distortion is held to a very low value.

II.   THE ULTRASONIC SIGNAL SOURCE

A.   DESIGNING THE SOURCE

1.   Circuit Configuration

Before transistors could be selected, a circuit
configuration had to be decided on. For high power stages with
low distortion there are two basic configurations used, both
with their advantages and disadvantages. These configurations
are complementary symmetry and quasi complementary symmetry.
The former uses complementary PNP - NPN pairs of transistors
in the output transistors, or both the drivers and an output
transistors. The advantages are that no inverter stage is
necessary and that there is a constant load on the driver stage
on both the positive and negative swings of the output voltage.
The latter is important in order to maintain linear operation
and low distortion. The basic disadvantages are procurement
of PNP and NPN transistors that are truly similar and the
necessity for matching the transistor pairs for optimum opera-
tion. The latter circuit configuration uses the same type
transistors (both either PNP or NPN) in the output stage but
drives them with a complementary pair of transistors in the
drive circuit. The advantages are that the output transistors
are easier to match and are of the same type. The disadvan-
tages are that the circuit is not as stable as pure complemen-
tary symmetry and distortion is higher because both halves of the
input sine wave do not see the same impedance at the drivers.
(Because one driver output pair is operated in a Darlington

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configuration and the other pair is operated in a common emitter driving an emitter follower - the former is higher in input impedance than the latter.) In addition, biasing requirements are complicated by the NPN - NPN pair having twice the V_{be} of the PNP - NPN pair (or the PNP - PNP having twice the V_{be} of the NPN - PNP depending of the type of output transistors used).

For this application, two stages of pure complementary symmetry were chosen: a NPN - PNP pair driving a PNP - NPN pair. The circuit appears in Figure 1 Appendix A. Rather than use diodes to compensate for the potential difference between the bases of the drivers caused by the V_{be} of these transistors as is customary in most circuits of this nature, it was decided to incorporate a transistor. (Whose V_{be} would be more nearly that of the drivers than the voltage across a diode.) The transistor was used in a regulated power supply circuit known as a "V_{be} multiplier" to provide the proper bias between the two bases of the transistors. In this circuit configuration, a bias can be obtained which can be tailored to what ever value is desired. The current through the transistor is chosen at a suitable value and collector-base and emitter-base resistors are chosen such that the base current is negligible compared to the current through these resistors, therefore the current through the collector-base resistor will be the same as that through the emitter-base resistor. The voltage across the emitter-base resistor, and therefore the current through it, is determined by the V_{be} of the transistor. Tailoring the ratio

of these two resistors will change the base-collector voltage and therefore the entire bias voltage. Here a potentiometer was used for complete control of the bias so that acrossover distortion could be minimized.

Various forms of predrivers were designed but testing found most of them unsatisfactory from a distortion standpoint. It was decided to feed the signal from a signal generator directly to the drivers and tailor the input impedance of the source to 600 ohms to match the output impedance of the signal generator. The signal generator was a Hewlett Packard Model 200AB which was fully capable of delivering up to 100 volts of signal if necessary. The input signal was coupled in with a normal R - C network, C_1 and R_1. In addition, some signal is bypassed around the bias supply to the voltage divider consisting of R_4 and R_5. This is to counter only bias changes caused by the unbypassed collector resistors R_8 and R_9 which might alter bias with signal.

To insure distortion free operation and stability, negative feedback was applied via a series R - C network comprised of C_4 and R_{10} from the output to the input. Since the source was capable of delivering large amplitude signals, a great deal of feedback was able to be applied to reduce distortion. In addition, since the feedback is of the shunt type, the additional benefit of further reduced output impedance was obtained.

Since a single ended power supply was used, a blocking capacitor was needed in the output. The capacitor was chosen

to have a very low impedance (.02 ohms reactive) at 15 kilohertz. This corresponded to 500 microfarads. In this way with the capacitive reactance virtually negligible, undesirable phase shifts and voltage drop were virtually eliminated until on the brink of complete short circuit. The output transistor emitter resistors are of lower value than is customary in power circuit such as this (the usual value being from .25 to 1 ohm). The value of .1 ohm was chosen because of the necessity for having low internal impedance. The series feedback from these resistors inherently increases the output impedance. The loss in temperature stability and susceptibility to thermal runaway were compensated for by the current limiting ability of the power supply, a Power Designs Incorporated Model 3650 R which had the ability to current limit at any value from 0 to 5.5 amperes. Thermal problems were compensated for by careful attention to physical mounting and heat sinking of the transistors.

2.   Selection of Output and Driver Transistors

For the output transistors, it was necessary to consider the expected continuous current drain on the source. Since this was on the order of five amperes the decision was made to use plastic silicon transistors for ease in mounting to a heat sink for good heat dissipation. Selection was also limited by availability in quantity (the process of matching requires a large stock to draw from). The resulting choices (MJE-2901 for the PNP and MJE-3055 for the NPN) were not the normally recommended complementary pair. However, their electrical similarity except for sign made them a likely prospect. A Tektronix type 576 curve tracer was used to examine the swept characteristics

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of the transistors in question. It was found that the dynamic

current gain, h_{fe}, of both types converged towards the same

value (about 30) at high currents - on the order of five amperes.

At lower quiescent currents it was found that the values from

device to device varied considerably. Therefore, it was in

this range, about 500 ma, that the output transistors were

matched. When matching, careful attention was also paid to the

slope of the curves (Emitter grounded, I_c vs. V_{ce} for stepped

I_b) beyond the knee (corresponding to $1/h_{oe}$) and also V_{be}

saturation voltage. The resulting pair were within 2% in all

three parameters and virtually identical save for polarity.

For the drivers, the ability to deliver moderate

currents, in the order of 200 milliamperes, and medium power

dissipation, at least one watt, prompted the choice of 2N 3053

for the NPN and 40319 for the PNP. Once again, availability

forced the use of not normally complemented devices. Great care

with the curve tracer was also needed here.

It was found that h_{fe} did not vary drastically with

increasing current carrying as in the output transistors, so

the drivers were matched in the range from 50 to 250 milliamperes.

As with the output transistors the parameters of h_{fe}, V_{be}

saturation voltage and $1/h_{oe}$ were matched as closely as possible.

The drivers were within the limits of the display accuracy of

the curve tracer to being perfectly identical, within 1%.

3. Physical Consideration

For best possible heat dissipation and maintainance

of close temperature similarity among the individual transistors

all transistors were mounted to a common heat sink. Number

6-32 bolts were used to mount the output transistors to the heat sink. A slight improvement in conduction of heat away from the transistor was obtained by drilling and threading the holes in the heat sink to accept the 6-32 bolt rather than use a nut to hold everything secure. Very fine mica washers were used for insulation between the transistors and the heat sink but liberal amounts of silicone grease were used on all surfaces to keep thermal conductivity as high as possible.

The drivers, having TO-5 cases, were a little more difficult to mount. A socket like clip which would hold these transistors in an inverted position was obtained. This clip had a 6-32 lug on its base so the heat sink was drilled and tapped to accept them, the threads and mount were coated with silicone grease before mounting to the large heat sink.

The driver transistors were then coated with silicone grease and pressed into the clips. The grease prevented air gaps, so detrimental to heat flow, from becoming a problem. The overall assembly was capable of dissipating over 100 watts with only 70° centigrade rise in temperature above ambient. In addition, constant temperature from device to device was virtually 100% sure due to such close proximity and good thermal conductivity. Figure 2, Appendix A shows this arrangement.

B.   TESTING THE SOURCE

1.   Distortion

The source was tested under various static loads of 5 ohms, 1 ohm and .2 ohms for distortion at maximum current, 5 amperes peak to peak. Distortion was checked by using a

Donner Wave Analyser Model 2102 to measure the amplitude of
the expected harmonics in the waveform in the ouput. 15 kilo-
hertz had to be used due to the limitation of 50 kilohertz on
the wave analyser. The amplitude of the second harmonic was
found to be .3% greater in the output than in the input waveform.
Third harmonic content was .5% greater in the output than in
the input at worse case.

    2.   Internal Impedance

Internal impedance was estimated by adjusting the
source for one ampere peak to peak of current at one ohm load.
The load was reduced to .2 ohm and the output voltage amplitude
noted (.55 volts peak to peak). The load was then increased
to 5 ohms and the output amplitude noted (1.2 volts peak to
peak).

From Ohm's law then, the output current is equal to
Internal impedance can then be determined by use of
Ohm's law, assuming that the drive received from the signal
generator is not altered by the change in load. The source
can then be reduced to its Thevinin equivalent. Figure 3
Appendix A.

From Ohm's law then, the output current is equal to
the Thevinin voltage (V) divided by the total circuit resistance
$(R_i + R_L)$ which is also equal to output voltage (V_o) divided
by load resistance:

$$\frac{V}{R_i + R_L} = \frac{V_o}{R_L}$$

Substituting several values of V_o and R_L and solving for V:

$$V = \frac{V_{o1}(R_i + R_{L1})}{R_{L1}} = \frac{V_{L2}(R_i + R_2)}{R_{L2}}$$

which solved for R_i yields:

$$R_i = -\frac{R_{Lo}\,R_{L2}\,(V_{o2} - V_{o1})}{R_{L2}\,V_{o1} - R_{L1}\,V_{o2}}$$

Substituting in the three values of R_L and V_o one gets an internal impedance of approximately .26 ohms.

From distortion and internal impedance data, the source was decided suitable for an actual ferroresonance study.

III. <u>THE SERIES RESONANT CIRCUIT</u>

A. OBSERVATION OF THE PHENOMENON

The core chosen for study was an Indiana General CF-101-F core. (See Figure 4 Appendix A) 30 turns of number 34 wire were placed on the core to form an inductor. A capacitor of .1 microfarads was placed in series with the inductor and a signal of approximately 20 kilohertz was placed across this combination. The waveform of the source was monitored to check for distortion and the amplitude of the signal was increased while watching the inductor voltage. As the voltage across the circuit was increased, the circuit passed into classical ferro-resonance where the capacitor voltage waveform approximated a square wave. Increasing the voltage further brought a period of instability followed by transition into another stable mode. It was in this second mode that the asymmetrical behavior was noted. The capacitor voltage contained a strong third harmonic term. The inductor voltage was also highly asymmetrical, minor changes in amplitude effecting variation in the dis-parity of the period of the two halves of the waveform. With proper amplitude it was possible to realize a ratio of 2 to 1 between the periods of the positive going "half" cycle and the negative going "half" cycle with a marginal stability. The most stable inductor waveform is that of Figure 1 Appendix B. It was noted that in transitioning from instability to stability, as the voltage amplitude reached that necessary for stability, the stable asymmetric mode would assume the waveform in one of

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two phases differing by 180°. Once one of the two phases was

assumed by the circuit it was maintained until the input

voltage amplitude was changed to take it out of the stable

range. In addition, resumption of stability brought on the two

phases in random fashion.

In order to pursue the study further it was desired to

examine the dynamic hysterisis loop of the core. A second

winding comprising five turns of number 30 wire was placed on

the core. This winding was terminated in an R - C integrator

consisting of a 100 kilohm resistor in series with a .5 micro-

farad capacitor. (Other values tried contained unbearable

integration errors) Since the series combination was nearly

totally resistive at the frequency in question, the voltage

across the pair was proportional to the current in the winding

which is proportional to magnetizing force. The voltage across

the capacitor, on the other hand, is proportional to the

integral of the winding current which is proportional to flux

density. Placing these voltages on the horizontal and vertical

channels respectively of an ANALAB Type 1100 two channel

ocilloscope, a B - H loop was obtained as can be seen in Figure

2 of Appendix B. There are still serious integration errors

in the loop, and the B - H loop shown is the optimum obtainable.

The problem lay in the capacitor voltage being of such a small

value compared to the voltage across the combination that the

high amplification necessary in the ocilloscope to display it

introduced distortion as well as delays in the waveform.

Increasing the capacitance to resistance ratio resulted in

errors due to improper integrator time constant for the frequency

used. Integrators using lower resistance values with high capa-
citances also excessively loaded the sense coil and reflected
losses back to the inductor winding which altered the "Q" of
the inductor. When this happened the waveform across the
inductor was degraded and the stability of the asymmetric wave-
form suffered. In spite of the integration errors present and
the degraded appearance of the B - H loop, one thing is quite
obvious, there is a serious anomaly present. The hysteresis
loop is definitely asymmetrical, there is a minor loop present.
The minor loop cannot be due to the hysteretic properties of
the core; however, it can be thought of as showing the history
of the magnetization of the core through one cycle of its
performance in the circuit under study. A close-up of the minor
loop can be seen in Figure 3 Appendix B. Rudenburg has shown
likely contours for minor loops during stable symmetrical modes
of ferroresonance which give rise to high order subharmonics
which can be seen in Figure 4 Appendix B. The loop present in
Figures 2 and 3 are very similar to this hypothetical loop in
spite of their degraded appearance. The asymmetry of the minor
loop can be seen.

B.   QUALITATIVE DESCRIPTION OF THE PHENOMENON

Taking the hypothetical hysteresis loop in Figure 6 and
the corresponding waveforms in Figure 7, a qualitative descrip-
tion of the phenomenon can be given. Points 1, 2, 3 and 4 on
the hysteresis loop correspond to t_1, t_2, t_3 and t_4 on the
voltage waveforms. Starting at t_1, the input voltage is positive
but decreasing, the capacitor voltage is relatively constant

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and is greater than the input voltage. The voltage across the coil is $V_{in} - V_c$, the polarity is tending to drive the core to negative saturation. As the drive voltage goes negative this effect is increased until the core passes into negative saturation. Prior to the input voltage reaching its negative maximum the core saturates. At time t_2 the capacitor voltage reverses polarity. This is due to two factors; the first being ferroresonant reversal of the inductor voltage, the second being the approaching arrival at negative maximum by the input voltage. The capacitor experiences the sum of these voltages and attempts to charge to nearly twice the input voltage through the low impedance of the saturated inductor. Once again the capacitor voltage exceeds input voltage and the inductor now sees a voltage of polarity tending to drive it toward positive saturation and now of a greater magnitude than that which drove it into negative saturation in the time from t_1 to t_2. This drive increases until the input voltage reaches negative maximum where it then begins to decrease. Up to this point the rate of switching toward positive saturation has been much more rapid than the rate toward negative saturation pre-viously, hence a more rapid arrival at positive saturation by the core. At t_3 the input waveform is reaching zero and at the same time the inductor is undergoing another ferroresonant reversal. The capacitor tries to charge to this voltage, but since the drive voltage is zero the charging rate is slower and since the core was not aided by the drive voltage the reversal is slower. Since the sides of the B - H loop are not vertical,

(indicating a less than infinite impedance) the capacitor sees
a finite impedance to discharge through. However, the rising
input voltage slows this discharge and the capacitor voltage
levels off. Now at t_4, the capacitor voltage exceeds the
input voltage, the core once again heads toward negative
saturation; however, before this process can get well underway
the input voltage exceeds the capacitor and the core tries to
change directions and move into positive saturation. Before
this process can reach completion the input voltage has
reached its positive maximum and began to decrease. It is
this chain of events after t_4 which account for the minor loop.
As the input voltage decreases below the capacitor voltage,
the cycle is completed and the loop finished. The cycle can
now begin again.

C.   QUANTITATIVE DESCRIPTION OF PHENOMENON

   1.   Circuit Equations

       By Faraday's law, the voltage across the coil is
equal to $N \frac{d\phi}{dt}$.

   The entire circuit is governed by the integrodifferential
equation:

$$N\frac{d\phi}{dt} + R_i + \frac{1}{C} \int i \, dt = - E \cos \omega t \tag{1}$$

where $\phi(0) = \phi_o$, $i(o) = 0$, $\omega = 2\pi \times 2.1 \times 10^4$ and where R is
the circuit impedance.

       In Reference 1, Dennard shows the development of an
equation for $i(t)$ from a plot of ϕ vs. i for the circuit. Two
points on the curve were used, $(6.2 \times 10^{-7}, 0.15)$ and $(2.0 \times$

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10^{-7}, 0.03). These were substituted to the general cubic equation:

$$i = b_1 \phi + b_3 \phi^3 \tag{2}$$

Simultaneous solution of the two resulting equations gave

$$i = 1.46 \times 10^5 \phi + 3.78 \times 10^{17} \phi^3 \tag{3}$$

$$b_1 = 1.46 \times 10^5 \qquad b_2 = 3.78 \times 10^{17}$$

After insertion of the values into the circuit equation and differentiation to remove the integral, the result using operator notation is:

$$D^2 + \frac{Rb_1}{N} D + \frac{b_1}{NC} \phi + \frac{Rb_3}{N} D + \frac{b_3}{NC} \phi^3 = \frac{\omega E}{N} \sin \omega t \tag{4}$$

To separate the constant flux component, ϕ_o, a new variable is defined:

$$x(t) = \phi(t) - \phi_o \tag{5}$$

This change of variable allows (4) to be rewritten as:

$$(D^2 + K_1 D + K_2)X + 3\phi_o (K_4 D + K_3)X^2 + (K_4 D + K_3)X^3 =$$

$$- K_5 + K_6 \sin \omega t \tag{6}$$

where

$$K_1 = \frac{Rb_1 + 3\phi_o^2 Rb_3}{N}$$

$$K_2 = \frac{b_1 + 3\phi_o^2 b_3}{NC}$$

$$K_3 = \frac{b_3}{NC}$$

$$K_4 = \frac{Rb_3}{N}$$

$$K_5 = \frac{b_3\phi_o^2 + b_1\phi_o}{NC}$$

$$K_6 = \frac{\omega E}{N}$$

From reference 1, ϕ_o is approximately 3.0×10^{-8} webers. The effect of ϕ_o upon the terms in which it is contained in powers greater than one is negligible, therefore, the following approximations are in order.

$$K_1 \approx \frac{Rb_1}{N} = 1.95 \times 10^4$$

$$K_2 \approx \frac{b_1}{NC} = 4.87 \times 10^{10}$$

$$K_3 \approx \frac{b_2}{NC} = 1.26 \times 10^{23}$$

$$K_4 \approx \frac{Rb_3}{N} = 5.05 \times 10^{16}$$

$$K_5 \approx \frac{b_1 \phi_o}{NC} = 1.5 \times 10^3$$

$$K_6 \approx \frac{E\omega}{N} = 2.2 \times 10^4$$

If the following changes are made:

$$a_1 = D^2 + K_1 D + K_2$$

$$a_2 = 3\phi_o (K_4 D + K_3) \quad .$$

$$a_3 = K_4 D + K_3$$

equation (6) can be rewritten as:

$$a_1 X + a_2 X^2 + a_3 X^3 = - K_5 - K_6 \sin \omega t \tag{7}$$

A solution for this equation in terms of $X(t)$ can be obtained by the method of reversion of series [Ref. 2].

Reversion of series is a method of solving ordinary nonlinear differential equations. The method is based on the algebraic procedure in reversing power series.

The nonlinear differential equation can be written as (in the notation of Reference 2):

$$a_1 y + a_2 y^2 + a_3 y^3 + \ldots = K \phi(t) \tag{8}$$

where t is the independent variable, y is the desired dependent variable, K is a constant, and $\phi(t)$ is a given function. The a_i coefficients are, in general, operators.

Assuming a solution of the form:

$$y = A_1 K + A_2 K^2 + A_3 K^3 + \ldots \tag{9}$$

The Ai's are determined by substituting (9) into (8) and equating coefficients of equal powers of K. This yields:

$$A_1 = \phi t / a_1 \tag{10}$$

$$A_2 = -a_2 A_1^2 / a_1 \tag{11}$$

$$A_3 = -(1/a_1)(2a_2 A_1 A_2 + a_3 A_1^3) \tag{12}$$

Equation (7), having been developed towards a solution by reversion of series, is in proper format if $K = 1$ and $\phi(t) = -K_5 + K_6 \sin \omega t$. The solution is then of the form:

$$X(t) = A_1(t) + A_2(t) + A_3(t) \tag{13}$$

Taking the derivative gives the voltage across the coil.

A_1, the initial term of (13), is found by solving the following equation for $A_1(t)$

$$a_1 A_1(t) = -K_5 + K_6 \sin \omega t \tag{14}$$

The operator a_1 appears in the solution of each A_i and is therefore the most influential element in the solution. The roots of a_1 are:

$$D = -\frac{K_1}{2} \pm \frac{j}{2} \sqrt{4K_2 - K_1^2}$$

Let: $\alpha = \dfrac{K_1}{2}$, $\beta = \dfrac{1}{2}\sqrt{4K_2 - K_1^2} \approx K_2$

$$\beta_o^2 = \alpha^2 + \beta^2 = K_2 \tag{15}$$

a_1 can then be rewritten as:

$$a_1 = D^2 + K_1 D + K_2 = (D + \alpha)^2 + \beta^2 \tag{16}$$

Taking the Laplace Transform of (14) yields:

$$A_1(s) = \frac{-K_5}{s[(s+\alpha)^2+\beta^2]} + \frac{K_6\,\omega}{(S^2+\omega^2)[(s+\alpha)^2+\beta^2]} \tag{17}$$

Direct transform pairs from Reference 3 gives:

$$A_1(t) = [\frac{1}{\beta_o^2} + \frac{1}{\beta_o\beta}\ e^{-\alpha t}\ \sin(\beta t - \psi_1)] +$$

$$K_6\omega\ \frac{1}{[(\beta_o^2 - \omega^2)^2 + 4\alpha^2\,\omega^2]}1/2 \quad [\frac{1}{\omega}\sin(\omega t - \psi_2)$$

$$+ \frac{1}{\beta}\ e^{-\alpha t}\ \sin(t - \psi_3)] \tag{18}$$

where $\psi_1 = \tan^{-1}\dfrac{\beta}{-\alpha} \approx \dfrac{\pi}{2}$

$$\psi_2 = \tan^{-1}\frac{2\alpha\omega}{\beta_o^2 - \omega^2} \approx 0$$

$$\psi_3 = \frac{-2\alpha\beta}{\alpha^2-\beta^2 + \omega2} \approx \pi$$

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$$A_1(t) \approx - K_1 + e^{-\alpha t} [K_1 \cos \beta t - \frac{K_2}{\beta} \sin \beta t] + \frac{K_2}{\omega} \sin \omega t$$

$$(19)$$

where $K_1 = 3.08 \times 10^{-7}$

$\quad K_2 = 9.28 \times 10^{-2}$

It can be noted that there appear exponential terms as well as the sine and cosine terms. Since only the steady state solution was of interest here, all exponential terms were excluded from the solution and do not appear in following expressions.

By similar attack, A_2 was evaluated and found to consist of terms of very small magnitude (the largest being 10^{-13}); therefore, A_2 was not considered necessary to the solution.

The defining equation for A_3 can be written:

$$a_1 A_3 = - a_3 [6\phi_o A_1 A_2 + A_1^3] \qquad (20)$$

Since it was determined previously that A_1 is greater than A_2, term for term, by several orders of magnitude. Therefor $6\phi_o A_1 A_2$ is less than A_1^3, term by term by an equivalent amount. Equation (20) can therefore be approximated by:

$$a_1 A_3 \approx - a_3 A_1^3 \qquad (21)$$

After removal of the decaying exponential terms, A_3 is found to be:

$$A_3 = 7.4 \times 10^{-8} - 1.5 \times 10^{-6} \sin (\omega t - .093) + 0.87 \times 10^{-8}$$
$$\sin 3 \omega t \tag{22}$$

A_1 after the removal of unnecessary exponential terms is:

$$A_1 = - 3.08 \times 10^{-7} + \frac{9.28 \times 10^{-2}}{\omega} \sin \omega t \tag{23}$$

The solution for the voltage across the inductor is obtained by taking the sum of equations (22) and (23), taking the derivative of the combination and then multiplying by the number of turns. The result is:

$$V_L = N[-8 \times 10^{-7} \omega \cos (\omega t - .0930) + 9.87 \times 10^{-9} (3\omega)$$
$$\cos 3\omega t] \tag{24}$$

Substitution of values into this formula through one cycle yields the waveform shown in Figure 5 Appendix B.

IV. <u>CONCLUSION</u>

A comparison of Figures 1 and 5 of Appendix B shows the analysis to be rather accurate. There are obviously higher order terms than third harmonic in the inductor wave form. The unequal size of the positive "humps" and also the inequality of the period of these two humps confirms this fact. Another factor causing inaccuracy may well be stray impedances (resistive), that could not be eliminated. Introduction of a .2 ohm resistor into the series coil-capacitor circuit caused an increase in this disparity; further decreasing the circuit impedance should cause the opposite to happen. This could be accomplished by further reduction in source impedance (which would be difficult with present semiconductor technology) or by larger wire for the coil winding, or by use of a higher quality capacitor with a much lower dissipation factor. Actually, a combination of all three would be necessary here since any one could only effect very minor changes (measured in hundredths of an ohm) in the overall circuit impedance.

In designing the source, the minimum specifications to be met were 2% distortion and less than .5 ohm internal impedance. The source as tested was 50% lower in both these categories.

All factors considered, the source performed very well, the standards of low distortion and low internal impedance were met and allowed a successful study of a complex phenomenon to be performed with good results.
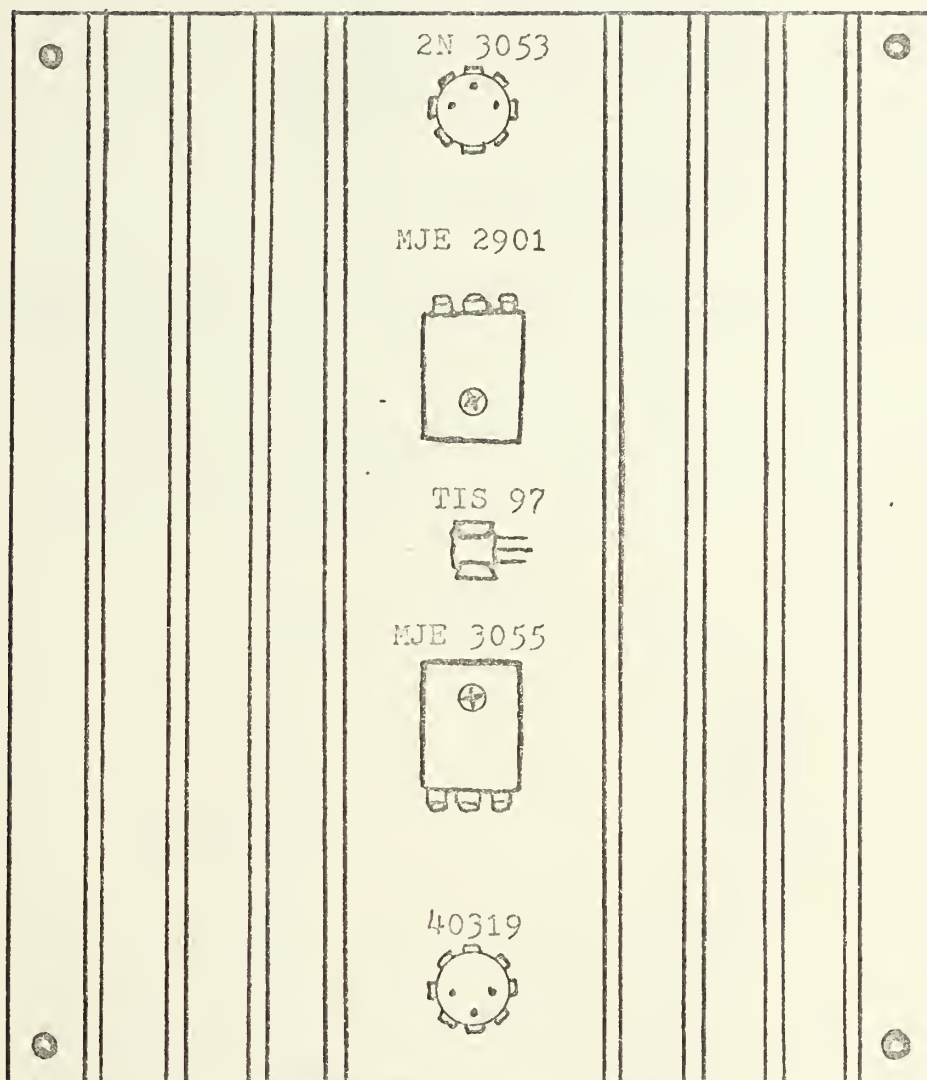
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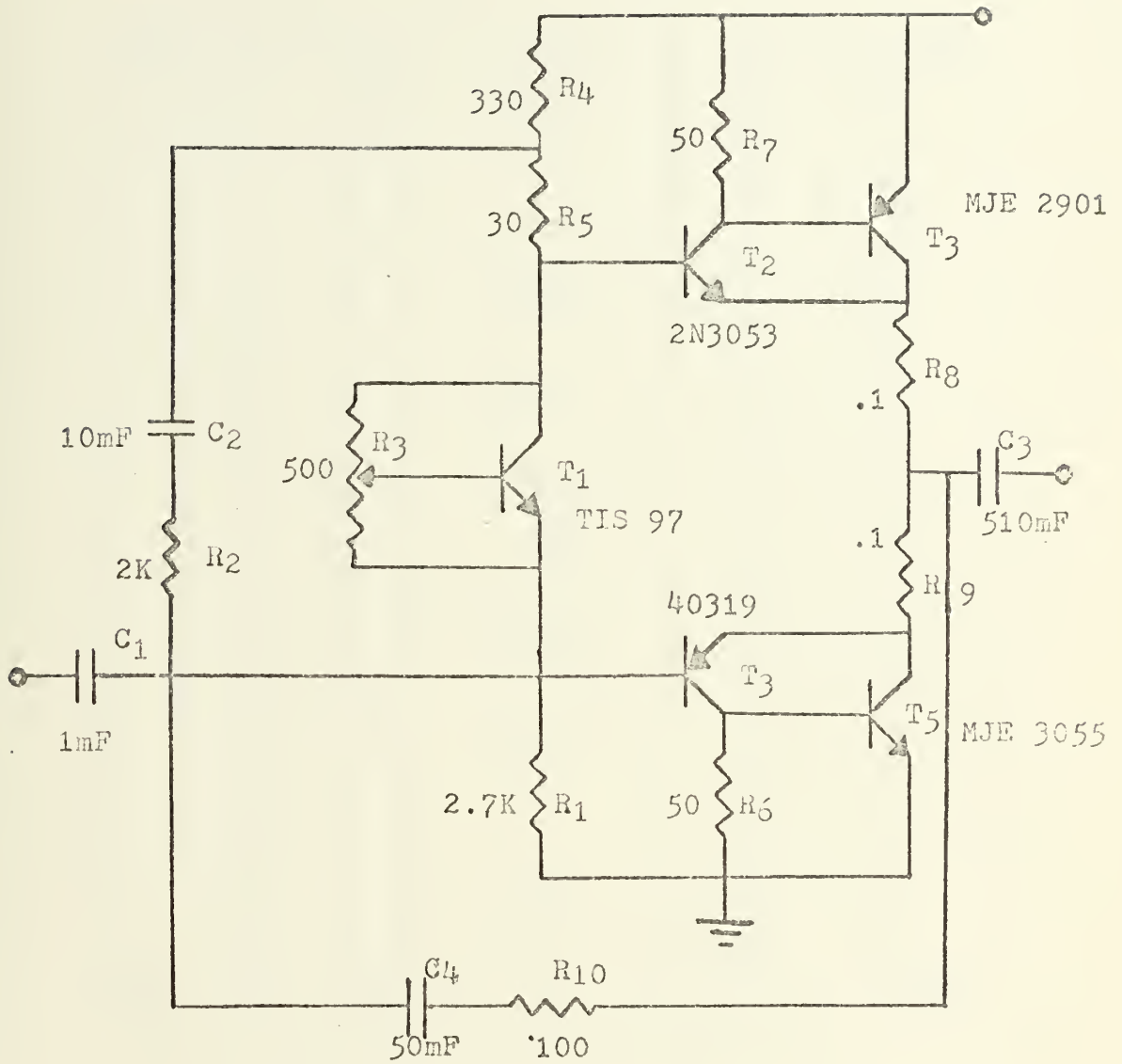
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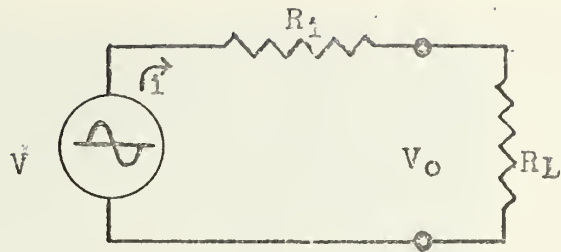
Figure 1

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SCALE : FULL

Physical mounting on heatsink

Figure 2
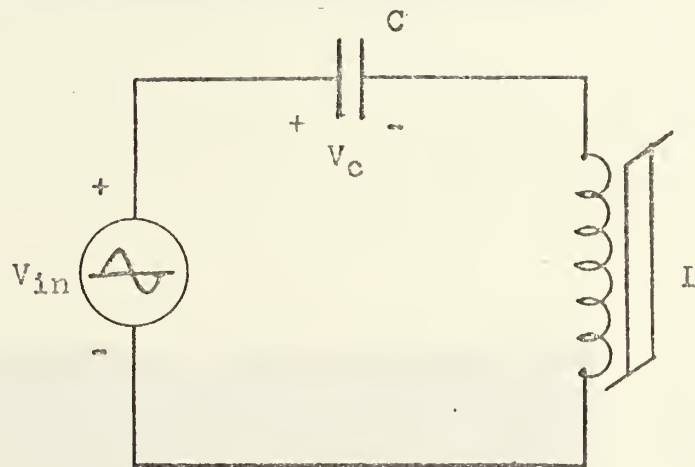
Thevenin equivalent of the Source for internal impedance
calculation.

Figure 3

Ferroresonant Circuit

$V_{in}=$ 0 to 10 volts pp   Core
      15 to 25 KHz
                                         I.D.= .12"

C = .1 microfarads   O.D.= .23"

L = 30 turns on Indiana General
      CF-101-F core   Ht.= .06"

                            Bs=3.4 KG   Br=1.47 KG

                            Hc= .18 Oe

Figure 4
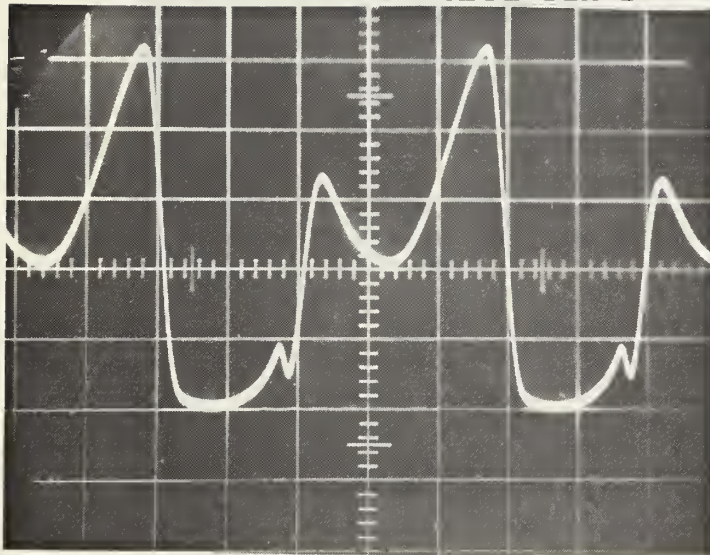
Inductor Waveform

Vertical

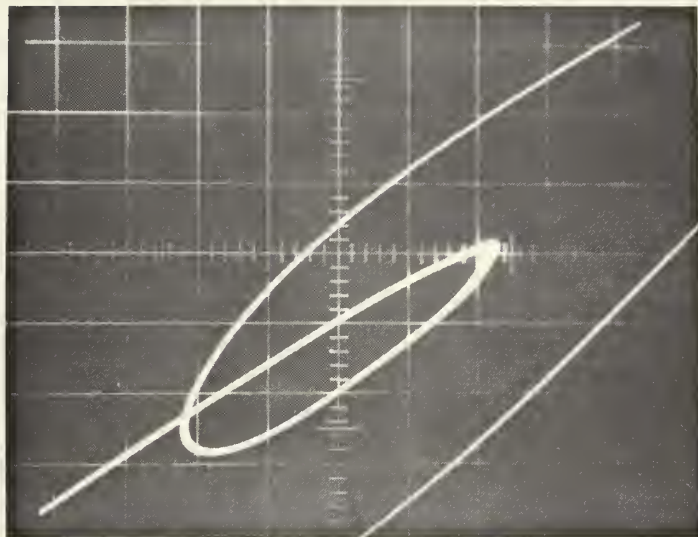.5 volts/cm

Horizontal

.05 msec/cm

Figure 1
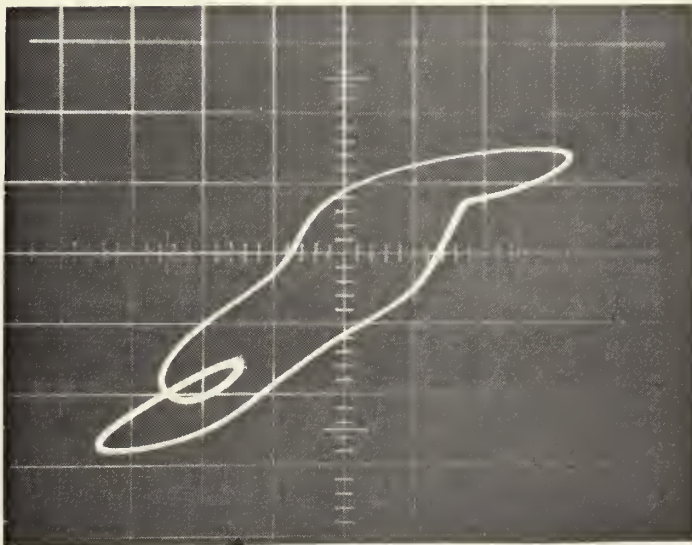
Hysteresis Loop

Vertical

2 mvolts/cm

Horizontal

10 ma/cm

Figure 2

Vertical = 1 mv/cm

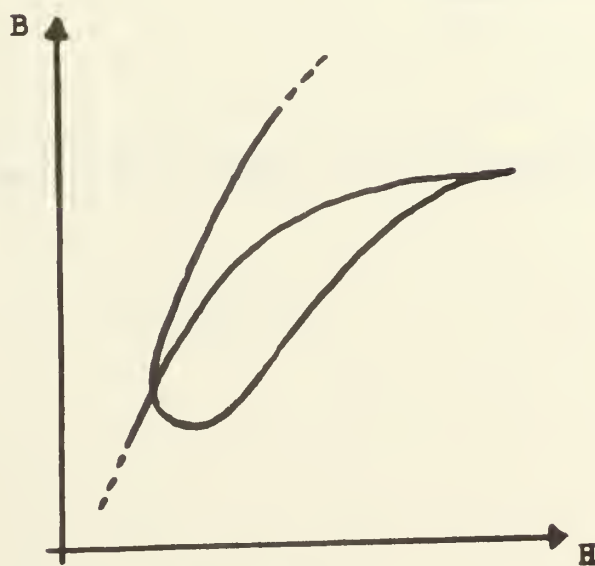Horizontal = 2. ma/cm

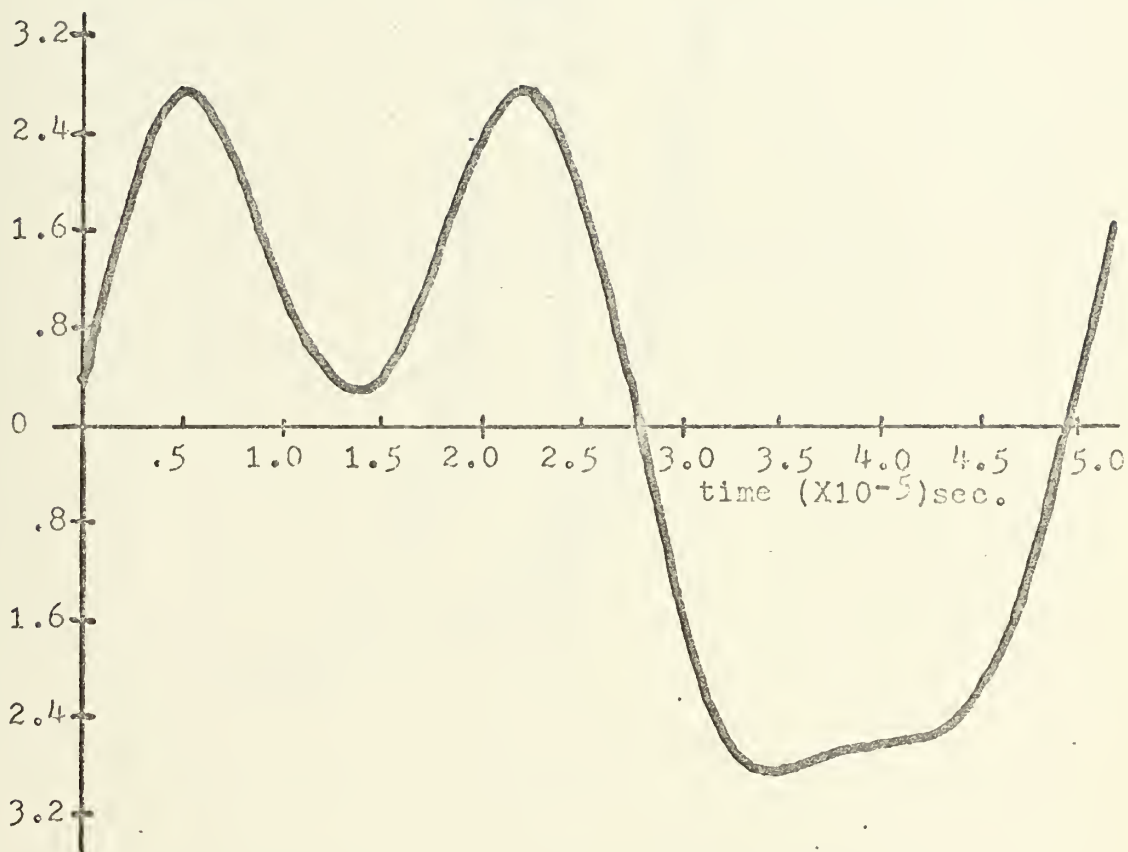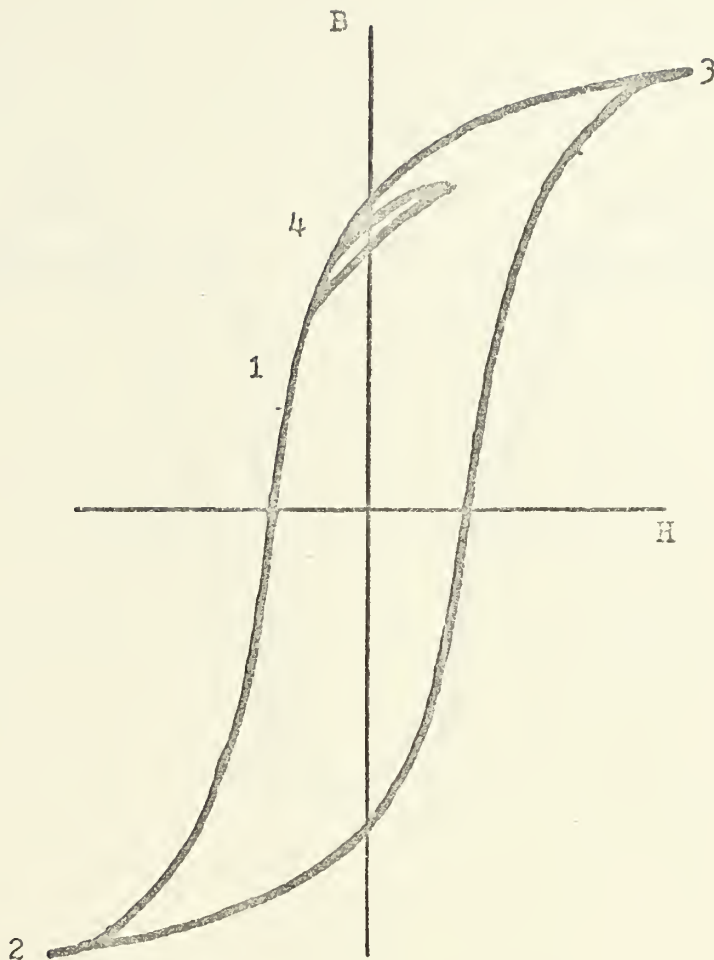Expansion of minor loop

Figure 3

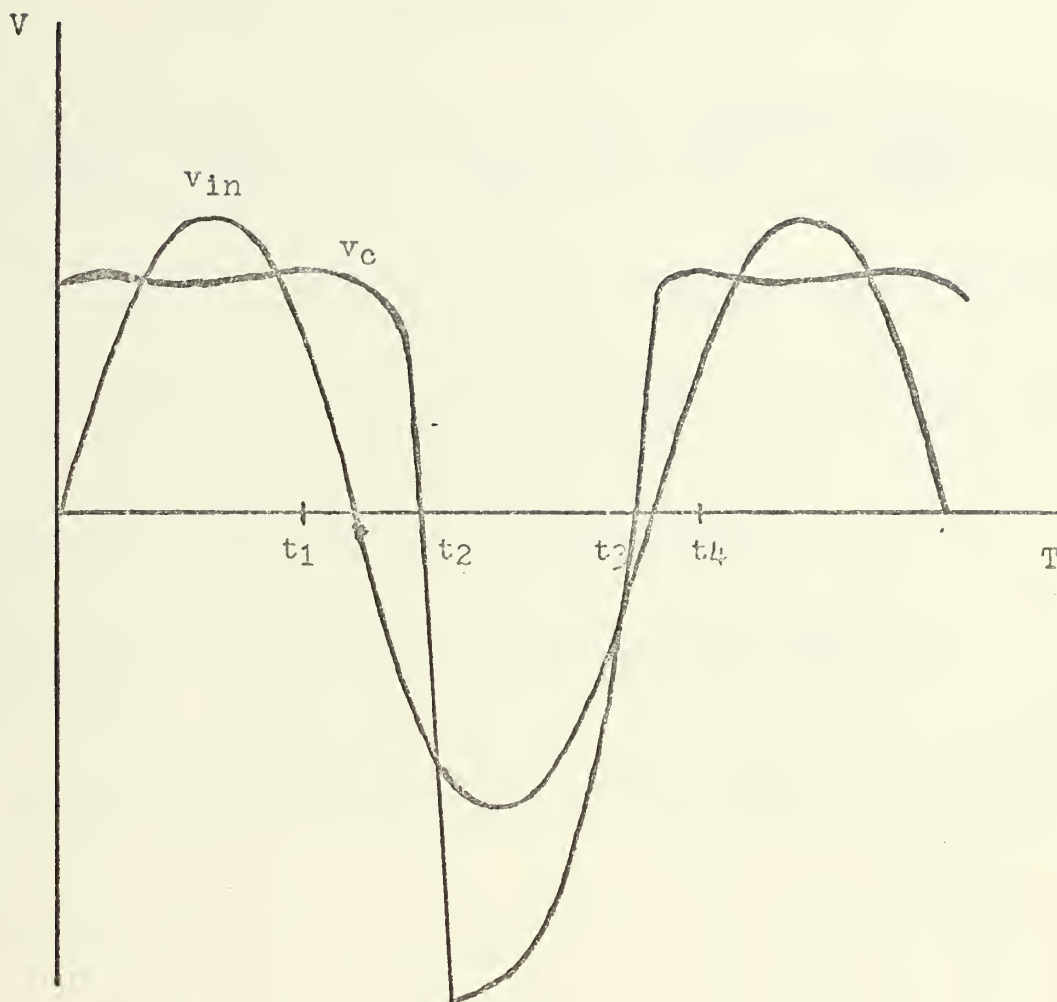Figure 4

Theoretical Waveform for Inductor

Figure 5

Hysteresis loop for qualitative description of the
ferroresonence phenomenon.

Figure 6

33

Input and capacitor voltages for qualitative
description of the ferroresonance phenomenon.

Figure 7

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DOCUMENT CONTROL DATA - R & D

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1 ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Naval Postgraduate School Monterey, California 93940	Unclassified
	2b. GROUP

3 REPORT TITLE

A Low Impedance High Current Ultrasonic Signal Source for Ferroresonance Study

4 DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; September 1970

5. AUTHOR(S) (First name, middle initial, last name)

John Reynolds Pratchios

6 REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1970	37	6

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

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13. ABSTRACT

In ferroresonant mode, a transformer is saturated and requires very high currents from the source. Previous studies of ferroresonance could not be performed effectively because of high internal impedance of the source. Introduction of transformers to improve these short-comings would have also introduced undesirable inductive effects. In this thesis, a transistor voltage source with low internal impedance and high current delivering ability is designed and tested. The source is used to investigate an unsymmetrical mode of series resonance. Resonant circuit behavior is both qualitatively and quantitatively described.

DD FORM 1473 (PAGE 1)
1 NOV 65

S/N 0101-807-6811

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ferroresonance						
Transistor						
Amplifers						